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Mathematical models for the apparent masses of standing subjects exposed to vertical whole-body vibration

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Abstract

Linear lumped parameter models of the apparent masses of human subjects in standing positions when exposed to vertical whole-body vibration have been developed. Simple models with a single degree-of-freedom (d.o.f.) and with two (d.o.f.) were considered for practical use. Model parameters were optimised using both the mean apparent mass of 12 male subjects and the apparent masses of individual subjects measured in a previous study. The calculated responses of two (d.o.f.) models with a massless support structure showed best agreement with the measured apparent mass and phase, with errors less than 0.1 in the normalised apparent mass (i.e., corresponding to errors less than 10% of the static mass) and errors less than 5° in the phase for a normal standing posture. The model parameters obtained with the mean measured apparent masses of the 12 subjects were similar to the means of the 12 sets of parameters obtained when fitting to the individual apparent masses. It was found that the effects of vibration magnitude and postural changes on the measured apparent mass could be represented by changes to the stiffness and damping in the two (d.o.f.) models.

1. Introduction

When the human body is supported by a structure, vibration of the structure may excite the dynamic response of the human body and the characteristics of the dynamic response of the body may influence the vibration of the structure supporting the body. An example is the dynamic interaction between the seated human body and a car seat: vibration at the floor of the car is amplified most at frequencies near the principal resonance frequency of the driving-point

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mechanical impedance, or apparent mass, of the human body due to the dynamic interaction between the human body and the seat cushion [1]. In this application, knowledge of the mechanical impedance, or apparent mass, of the human body and knowledge of the dynamics of the seat cushion can be used to predict the vibration transmission through the seat [2,3].

Dynamic interactions between the human body in a standing position and the structure supporting the body are also known, although less well considered than those for seated persons. In structures such as pedestrian bridges, sports halls, grandstands at sports stadiums, the movement of the human when walking, running or jumping can exert dynamic loads on the structure. The nature of the dynamic loads has been investigated and summarised by Bachmann and Ammann [4]. Once such dynamic loads or other sources induce a vibration of the structure, the passive human body can present an effective mass about 50–100% greater than its static mass [5]. The effect of the passive human body on the structure may therefore be underestimated by representing the human body as a rigid mass. This could result in unexpectedly high stresses in the elements of the structure affecting its integrity, or excessive vibration that might degrade its serviceability [4]. Therefore, when designing structures in which people may perform a variety of activities in addition to standing, knowledge of the dynamic behaviour of the human body at the driving-point and knowledge of the interaction between the body and the structure may be required.

The main objective of this study was to develop a theoretical expression of the driving-point apparent mass of the human body in standing postures. To assist practical applications, simple linear lumped parameter models were considered. Alternative lumped parameter models were developed from the apparent masses measured at various vibration magnitudes and in several standing postures in a previous experiment [5]. A similar study has been conducted by Wei and Griffin for seated subjects [6]. The effects of walking, running, or jumping on the supporting structure, as discussed by Bachmann and Ammann [4], were not included within the scope of this study. It was not the intention to model the internal movements of the body responsible for the observed characteristics of the apparent mass, such models are much more complex than is necessary for predicting the driving-point apparent mass of the human body.

2. Experimental results

An experiment in which the apparent masses of standing subjects were measured was conducted by the authors at the Institute of Sound and Vibration Research of the University of Southampton [5]. The experiments were conducted with a 1-m stroke electro-hydraulic vibrator. Twelve male subjects took part in the experiment; the mean age, height and weight of the subjects, together with the corresponding standard deviation, are presented in Table 1. The subjects were exposed to random vertical vibration in the frequency range between 0.5 and 30 Hz at five vibration magnitudes between 0.125 and 2.0 ms^{-2} r.m.s. Three standing postures were used in the experiment: a normal standing posture, a legs bent posture, and a one leg posture: the definitions of each posture are given in Table 2. Two vibration magnitudes, 0.25 and 1.0 ms^{-2} r.m.s., were used with the one leg posture, while five vibration magnitudes were used for the other two postures. The force at the interface between the vibrating floor and each subject was obtained with

Table 1
Mean and standard deviation of age, height and weight of subjects in the experiment (SD: standard deviation) [5]

Age (yr)		Height (m)		Weight (kg)	
Mean	SD	Mean	SD	Mean	SD
28.3	3.23	1.78	0.0553	73.9	7.57

Table 2
Definitions of the three standing postures used in the experiment

Posture	Definition
Normal standing	Keep the legs straight and locked with comfortable and upright upper-body and with 0.3 m separation between the feet
Legs bent	Hold the legs bent so that the knees were vertically above the toes with comfortable and upright upper-body and with 0.3 m separation between the feet
One leg	Stand on the left leg being straight with comfortable and upright upper-body

a force platform, Kistler 9281B, rigidly mounted on the shaker platform. The acceleration at the floor was measured with a piezo-resistive accelerometer, Entran EGCSY-240D*-10.

The apparent mass, $M(f)$, was calculated by dividing the cross spectral density function between the input acceleration and the force at the floor, $S_{af}(f)$, by the power spectral density function of the input acceleration, $S_a(f)$:

$$M(f) = S_{af}(f)/S_a(f). \tag{1}$$

The effect of the mass of the top plate of the force platform was eliminated by subtracting the apparent mass measured without a subject, ideally a constant modulus with zero phase at all frequencies, from the apparent masses measured with subjects.

A large variability in the apparent masses of subjects was partly attributed to their different static masses, as in previous studies with seated subjects (e.g. Ref. [7]). Hence, each apparent mass was 'normalised' by dividing it by the measured value of the apparent mass at the lowest frequency, 0.5 Hz, which was almost equal to the static mass of the subject.

$$M_n(f) = M(f)/M(0.5\,\mathrm{Hz}). \tag{2}$$

The normalised apparent mass helps focus on the dynamic characteristics of subjects (e.g. the amplification of the response of the body at the resonance frequency compared with that of a rigid mass).

3. Description of apparent mass models

The apparent masses of standing subjects obtained in previous studies appeared to be similar to the apparent mass of either a single d.o.f. system or a two d.o.f. system over the frequency range between 0 and 30 Hz, analogous to the apparent masses of seated subjects [5, 7]. In the measured apparent mass for the normal standing posture, there was one peak at about 5 Hz and, for most subjects, one broad peak in the frequency range between 10 and 15 Hz. With a 'legs bent posture' and a 'one leg posture', the first peak frequency in the apparent mass decreased to about 3 Hz and 4 Hz, respectively. Wei and Griffin [6] investigated four alternative lumped parameter models, either with a single d.o.f. or with two d.o.f., to represent the apparent mass of seated subjects exposed to vertical vibration. The four models used by Wei and Griffin, together with two similar models, were included in this study so as to investigate whether such models could also represent the apparent masses of standing subjects.

3.1. Single d.o.f. models

The single d.o.f. models used in this paper are shown in Figs. 1(a) (Model 1a) and (b) (Model 1b). The difference between Model 1a and b was that Model 1a had a massless support at the bottom, whereas the bottom structure in Model 1b had a mass m_0. A support mass of m_0 was incorporated in the model so that it could be applied to the construction of a mechanical model, or a dummy, for mechanical testing. The apparent mass of the two models in a stationary state can be given theoretically in complex functions by using the mass, stiffness and damping shown in Fig. 1:

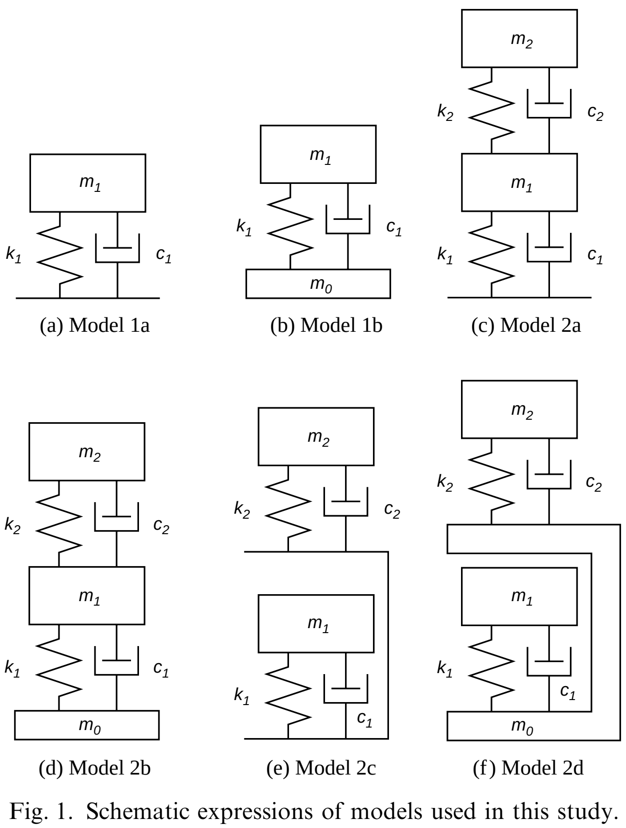

Fig. 1. Schematic expressions of models used in this study.

$$\text{Model 1a}: M_{1a}(\mathrm{i}\omega) = \frac{m_1(\mathrm{i}c_1\omega + k_1)}{(-m_1\omega^2 + \mathrm{i}c_1\omega + k_1)}, \tag{3}$$

$$\text{Model 1b}: M_{1b}(\mathrm{i}\omega) = M_{1a}(\mathrm{i}\omega) + m_0, \tag{4}$$

where ω is the angular frequency ($\omega = 2\pi f$) and $\mathrm{i} = \sqrt{-1}$.

3.2. Two d.o.f. models

The four types of two d.o.f. models used in this study are shown in Figs. 1(c) to (f) (Model 2a–2d). Model 2a and b had two mass–spring–damper systems in series (Figs. 1(c) and (d)), while Model 2c and d had two mass–spring–damper systems in parallel (Figs. 1(e) and (f)). The support structure in Model 2a and c had no mass, whereas the support structure in Model 2b and d had a mass of m_0. The support mass m_0 might be required to develop a mechanical dummy representing the apparent mass of the standing body for experimental testing. The apparent masses in a stationary state can be expressed by using masses, stiffnesses and damping shown in Fig. 1 for each model:

$$\text{Model 2a}: M_{2a}(\mathrm{i}\omega) = \frac{(\mathrm{i}c_1\omega + k_1)\{m_1(-m_2\omega^2 + \mathrm{i}c_2\omega + k_2) + m_2(\mathrm{i}c_2\omega + k_2)\}}{\{-m_1\omega^2 + \mathrm{i}(c_1 + c_2)\omega + (k_1 + k_2)\}(-m_2\omega^2 + \mathrm{i}c_2\omega + k_2) - (\mathrm{i}c_2\omega + k_2)^2}, \tag{5}$$

$$\text{Model 2b}: M_{2b}(\mathrm{i}\omega) = M_{2a}(\mathrm{i}\omega) + m_0, \tag{6}$$

$$\text{Model 2c}: M_{2c}(\mathrm{i}\omega) = \frac{m_1(\mathrm{i}c_1\omega + k_1)}{(-m_1\omega^2 + \mathrm{i}c_1\omega + k_1)} + \frac{m_2(\mathrm{i}c_2\omega + k_2)}{(-m_2\omega^2 + \mathrm{i}c_2\omega + k_2)}, \tag{7}$$

$$\text{Model 2d}: M_{2d}(\mathrm{i}\omega) = M_{2c}(\mathrm{i}\omega) + m_0. \tag{8}$$

3.3. Model parameter identification

The apparent masses calculated by Eqs. (3)–(8) for each of six models were compared with the apparent masses of standing subjects obtained in the previous experimental study mentioned in Section 2 [5], so as to determine model parameters. The model parameters were optimised to minimise the following error function:

$$err = \sum_n |M_m(n\Delta f) - M_c(n\Delta f)|^2, \tag{9}$$

where M_m is the measured apparent mass using complex numbers, M_c is the calculated apparent mass using complex numbers, Δf is the frequency resolution of the measured data (i.e., 0.25 Hz). In this study, frequencies between 0.5 and 20 Hz (where the peaks in the apparent mass were observed in the experimental data [5]) were used in the parameter identification. The frequencies are defined by the product of integers ($n = 2, 3, \ldots, 80$) and the frequency resolution, Δf, in Eq. (9). Optimised parameters were obtained by a non-linear parameter search method, based on the Nelder–Mead simplex method, provided within MATLAB (MathWorks Inc.).

The parameter identification method used in this study required initial values for each model parameter. The initial values of the natural frequencies for two d.o.f. models were selected as 3, 4, or 5 Hz for one of the mass–spring systems, depending on the posture, and 10 or 15 Hz for the second mass–spring system, based on inspection of the experimental data [5]. For single d.o.f. models, a natural frequency of 3–5 Hz was used as an initial value. It was deduced from an inspection of the experimental data that initial values appropriate for the damping ratios were 0.3–0.5. The models investigated in this study were not developed to model the internal movements of the body, so that the mass elements did not represent any particular body parts. Therefore, the initial values for the model masses were determined arbitrarily. The ratio of the two sprung masses, m_2/m_1, in the two d.o.f. models were selected between 0.5 and 2. For the mass of support structure, m_0, various initial values, with a maximum of 50% of the total mass of the model, were used.

The selection of initial parameters in the parameter search mentioned above resulted in nominally the same sets of optimised parameters for all models used in this study except for Model 2a and b. For Model 2a and b with two mass–spring–damper systems in series, the optimised parameters obtained when the lower initial natural frequency (i.e., 3–5 Hz) was assigned to the lower system (i.e., m_1 and k_1) were different from the optimised parameters obtained when the lower initial natural frequency was assigned to the upper system (i.e., m_2 and k_2). The former optimised parameters gave a greater difference between the apparent mass measured in the experiment and the apparent mass calculated from the model than the latter optimised parameters. The data presented in this paper are the parameters that minimised the maximum error in the modulus of the apparent mass at frequencies below 20 Hz, having obtained several sets of optimised parameters with different initial parameters.

4. Apparent mass model for normal standing posture

4.1. Models of mean responses

A model that represents the apparent mass of subjects when standing normally was developed using the apparent masses of 12 subjects exposed to random vibration at a magnitude of 1.0 ms^{-2} r.m.s. as measured in the experiment [5].

Sets of optimised parameters obtained for each of the models shown in Fig. 1 are tabulated in Table 3. The mean normalised apparent mass was used in the parameter identification, so the mass parameter shown in Table 3 has no units. Correspondingly, the units of the stiffness and damping parameters, based on SI units [Nm^{-1}] and [Nsm^{-1}], respectively, were divided by the unit of mass [kg] as in Table 3. Nominal corresponding parameters for a specific static mass of the body may be obtained by multiplying the parameters shown in Table 3 by the static mass. For the models with a support structure having a mass m_0 (Models 1b, 2b and 2d), the optimisation resulted in the mass parameter m_0 being less than 1% of the total mass in the parameter identification described above, so Model 1b, 2b and 2d were almost identical to Models 1a, 2a and 2c, respectively. This suggests that Models 1b, 2b and d are not necessary. However, so as to investigate the effects of the support mass on the model performance, in the parameter sets shown in Table 3, 10% of the sum of the other masses, $0.1m_1$ for Model 1b or $0.1(m_1 + m_2)$ for Model 2b and d, was assigned to m_0. The other parameters in these models were then obtained from the parameter optimisation procedure described in Section 3.3. The apparent masses and phases

Table 3
Optimised model parameters of all model types for the mean normalised apparent masses of 12 subjects in a normal standing posture

	Stiffness ($Nm^{-1}kg^{-1}$)		Damping ($Nsm^{-1}kg^{-1}$)		Mass (no unit)		
	k_1	k_2	c_1	c_2	m_0	m_1	m_2
Model 1a	1.34×10^3	—	5.16×10^1	—	—	1.03×10^0	—
Model 1b	1.30×10^3	—	4.31×10^1	—	9.55×10^{-2}	9.55×10^{-1}	—
Model 2a	4.39×10^3	5.53×10^2	3.71×10^1	1.18×10^1	—	5.74×10^{-1}	3.94×10^{-1}
Model 2b	4.39×10^3	5.96×10^2	2.16×10^1	1.38×10^1	8.94×10^{-2}	4.58×10^{-1}	4.37×10^{-1}
Model 2c	2.37×10^3	8.49×10^2	2.48×10^1	1.65×10^1	—	3.45×10^{-1}	6.33×10^{-1}
Model 2d	1.82×10^3	8.93×10^2	1.42×10^1	1.76×10^1	9.09×10^{-2}	2.54×10^{-1}	6.55×10^{-1}

Nominal corresponding parameters for a specific static mass may be obtained by multiplying the parameters shown by the static mass.

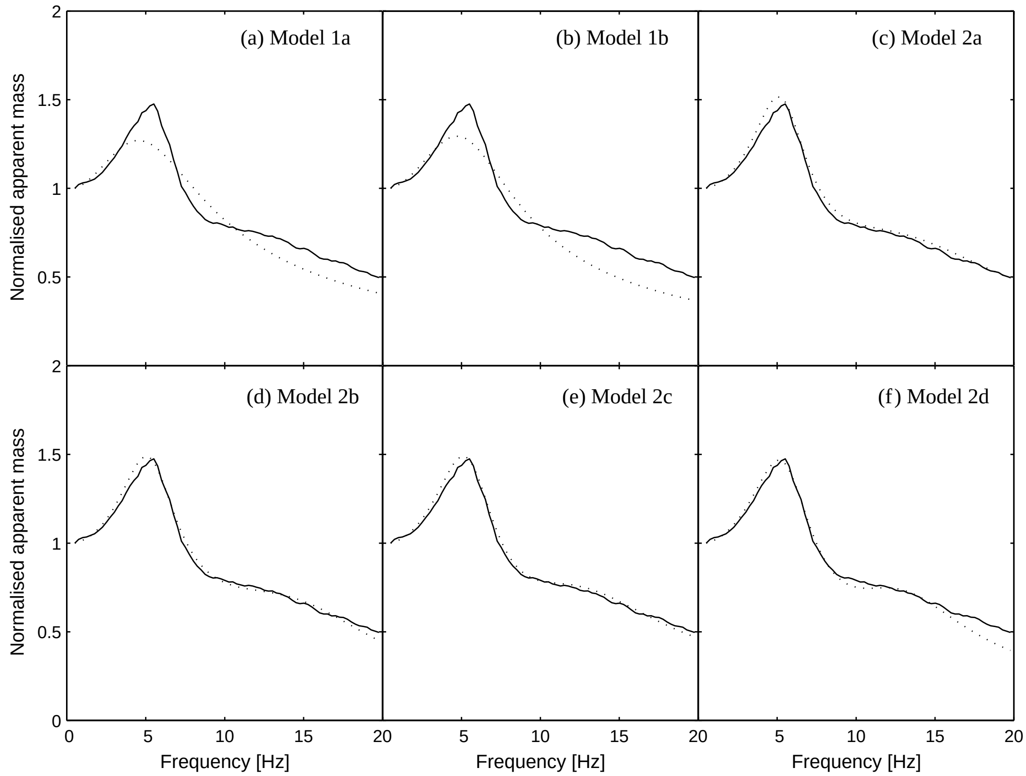

Fig. 2. Mean normalised apparent masses of 12 subjects in normal standing posture and the corresponding model responses for each model: ——, experiment [5]; - - - -, model.

calculated with the optimised parameters shown in Table 3 are presented in Figs. 2 and 3 with the corresponding experimental data.

As seen in Figs. 2 and 3, the experimental data showed the characteristics of a two d.o.f. system in the frequency range below 20 Hz, so the model responses calculated from all two d.o.f. models

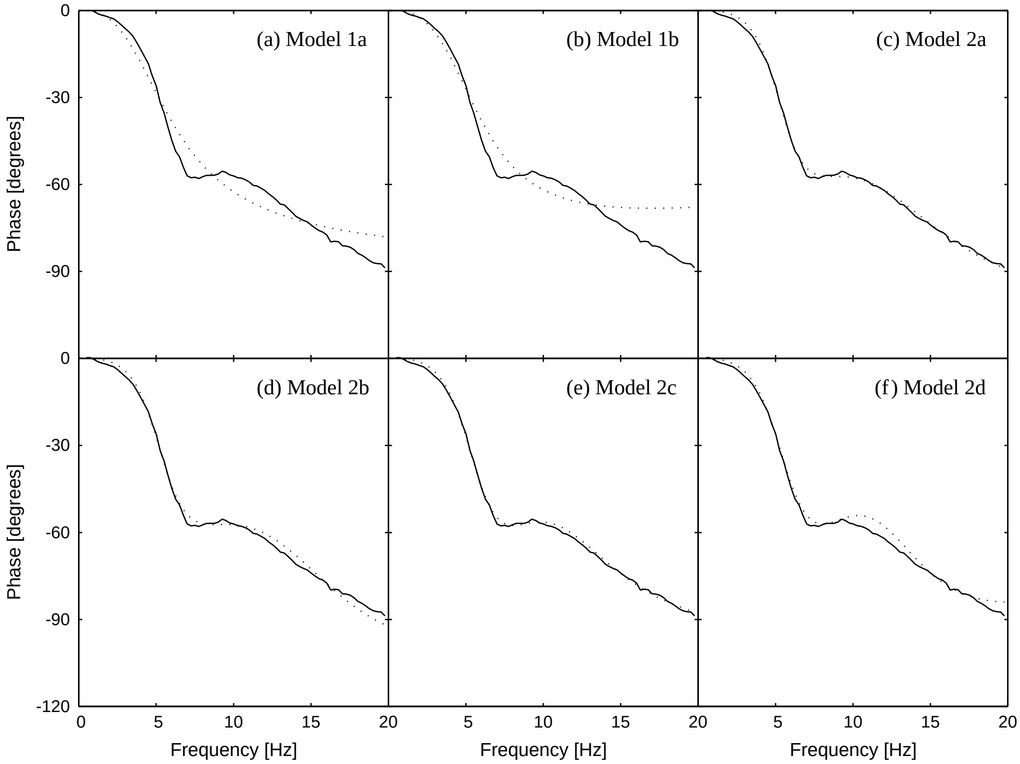

Fig. 3. Mean phase of the normalised apparent masses of 12 subjects in normal standing posture and the corresponding model responses for each model: ——, experiment; - - - -, model.

(Model 2a–d) fitted the experimental data better than the single d.o.f. models (Model 1a and b). The maximum differences, or errors, in the normalised apparent mass and phase between the experimental data and the model responses were obtained over the frequency ranges from 0.5 to 10 Hz and from 0.5 to 20 Hz, as presented in Fig. 4. The maximum errors in the normalised apparent mass and phase were greater than 0.2 and 10°, respectively, for the two single d.o.f. models. For the two d.o.f. models, the maximum errors in the normalised apparent mass and phase were less than about 0.1 and 5°. The two d.o.f. models with a massless structure (Model 2a and c) showed a better agreement with the experimental data than those with a support having a mass of m_0 (Model 2b and d), especially in the frequency range between 10 and 20 Hz.

4.2. Models of individual responses

Models of the apparent masses of individual subjects were developed using the two models (Model 2a and c as presented in Fig. 1) that showed the best agreement with the mean experimental data, as described in the previous section. The apparent masses measured with 12 individual subjects at a vibration magnitude of 1.0 ms^{-2} r.m.s., the same condition used to illustrate the mean data in the previous section, were used for determining model parameters

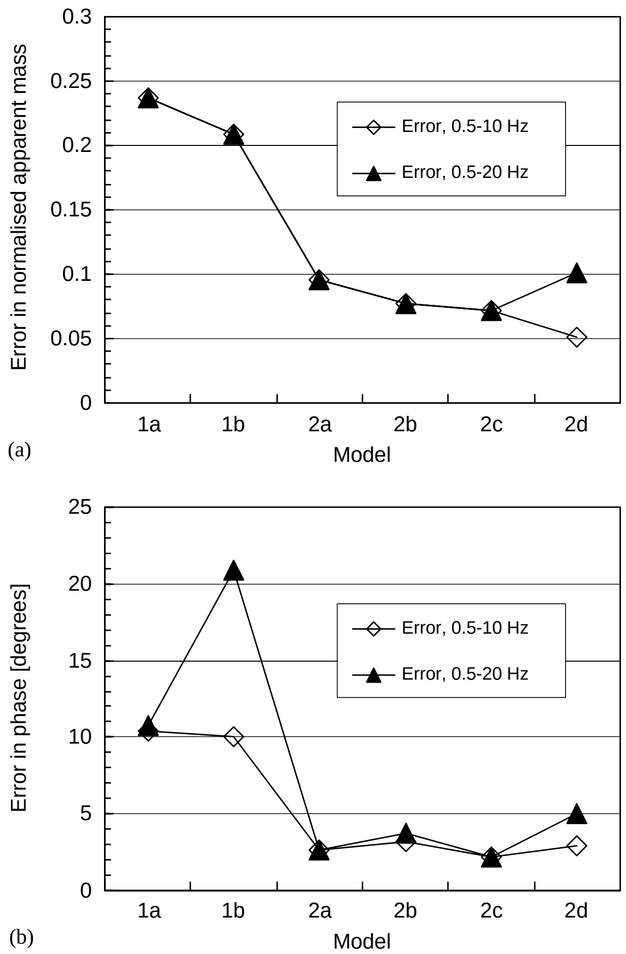

Fig. 4. Maximum errors in (a) normalised apparent mass and (b) phase between the experimental data and the responses calculated from each model in the frequency range below 10 Hz and below 20 Hz for normal standing posture.

(i.e., the mean data in the previous section were obtained from the individual data presented in this section).

The optimised model parameters for individual subjects are presented in Table 4 for Model 2a and in Table 5 for Model 2c. In Tables 4 and 5, the parameters given in rows titled ‘mean’ are the arithmetic average of the parameters for the 12 subjects. The parameters in a row titled ‘mean model’ are the parameters shown in Table 3, which were obtained by fitting the model response with the mean normalised apparent mass, multiplied by the average total mass of individual models shown in Tables 4 and 5.

Comparisons between the calculated apparent masses and phases for Model 2a with the parameters shown in Table 4 and the individual experimental data are made in Figs. 5 and 6. Figs. 5 and 6 indicate that Model 2a represented individual responses with very small errors. Model 2c with the parameters shown in Table 5 also showed good agreement with each set of

Table 4
Optimised model parameters of Model 2a for the apparent masses of 12 individual subjects in a normal standing posture

	Stiffness (Nm^{-1})		Damping (Nsm^{-1})		Mass (kg)	
	k_1	k_2	c_1	c_2	m_1	m_2
Subject 1	3.21×10^5	4.22×10^4	3.05×10^3	5.86×10^2	50.9	22.7
Subject 2	2.29×10^5	2.32×10^4	3.37×10^3	8.51×10^2	35.9	26.7
Subject 3	2.81×10^5	3.36×10^4	3.23×10^3	6.34×10^2	43.7	21.8
Subject 4	3.69×10^5	4.30×10^4	2.52×10^3	1.04×10^2	36.1	29.3
Subject 5	3.20×10^5	4.97×10^4	2.89×10^3	9.38×10^2	50.9	33.0
Subject 6	3.70×10^5	4.23×10^4	1.90×10^3	7.44×10^2	37.4	25.8
Subject 7	2.97×10^5	4.08×10^4	2.87×10^3	6.99×10^2	46.2	28.9
Subject 8	2.84×10^5	3.06×10^4	3.19×10^3	3.91×10^2	45.4	19.7
Subject 9	3.04×10^5	4.75×10^4	2.08×10^3	8.51×10^2	50.9	28.9
Subject 10	3.40×10^5	2.37×10^4	3.27×10^3	7.04×10^2	42.8	26.6
Subject 11	3.55×10^5	3.98×10^4	2.51×10^3	9.89×10^2	34.4	32.5
Subject 12	2.39×10^5	3.86×10^4	1.99×10^3	6.91×10^2	50.9	27.9
Mean	3.09×10^5	3.79×10^4	2.74×10^3	7.60×10^2	43.8	27.0
Mean model	3.21×10^5	4.04×10^4	2.71×10^3	8.63×10^2	42.0	28.8

Table 5
Optimised model parameters of Model 2c for the apparent masses of 12 individual subjects in a normal standing posture

	Stiffness (Nm^{-1})		Damping (Nsm^{-1})		Mass (kg)	
	k_1	k_2	c_1	c_2	m_1	m_2
Subject 1	2.22×10^5	6.68×10^4	2.34×10^3	8.69×10^2	33.2	40.9
Subject 2	6.66×10^4	1.42×10^4	3.35×10^3	3.09×10^2	46.2	16.8
Subject 3	1.86×10^5	5.10×10^4	2.49×10^3	8.84×10^2	29.7	36.1
Subject 4	1.59×10^5	7.19×10^4	1.34×10^3	1.62×10^3	18.4	47.8
Subject 5	1.73×10^5	7.55×10^4	1.90×10^3	1.26×10^3	29.1	55.5
Subject 6	2.08×10^5	6.55×10^4	1.41×10^3	1.09×10^3	23.4	41.2
Subject 7	1.89×10^5	5.84×10^4	2.11×10^3	9.34×10^2	29.5	46.1
Subject 8	2.31×10^5	4.34×10^4	2.71×10^3	5.39×10^2	33.7	31.5
Subject 9	1.36×10^5	9.07×10^4	9.85×10^2	1.50×10^3	22.3	59.1
Subject 10	2.11×10^5	3.85×10^4	2.16×10^3	1.22×10^3	28.0	42.1
Subject 11	1.68×10^5	5.58×10^4	1.76×10^3	1.26×10^3	21.5	46.1
Subject 12	1.27×10^5	6.60×10^4	1.21×10^3	1.06×10^3	26.5	53.5
Mean	1.73×10^5	5.81×10^4	1.98×10^3	1.05×10^3	28.5	43.1
Mean model	1.73×10^5	6.21×10^4	1.81×10^3	1.21×10^3	25.2	46.3

individual data, although this is not illustrated in this paper. For both models, the differences in the apparent mass between the model responses and the experimental data were less than 10% of the static mass for each individual, corresponding to a normalised apparent mass of 0.1. The error in the phase was less than 8°.

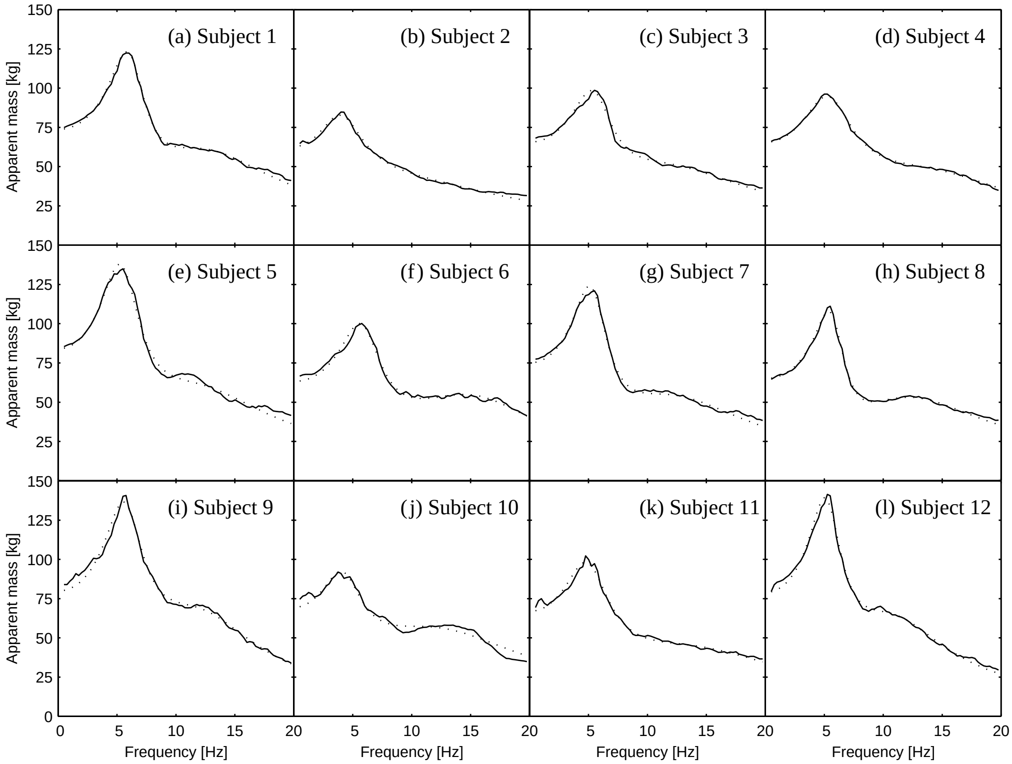

Fig. 5. Comparison between the measured apparent mass and the apparent mass of Model 2a with the parameters presented in Table 4 for 12 subjects in normal standing posture: ——, experiment; - - - -, model.

5. Models for different vibration magnitudes and postures

5.1. Effect of vibration magnitude on model parameters

In the experimental data, the frequency of the peaks in the apparent mass showed statistically significant decreases with increasing magnitude of the vibration [5]. Mathematical models were therefore developed for the apparent mass measured at the vibration magnitudes used in the experiment. The optimised model parameters for Model 2a and c, obtained for the mean measured normalised apparent mass of the 12 subjects in the normal standing posture at 0.25, 0.5, 1.0 and 2.0 $\mathrm{ms^{-2}}$ r.m.s. are tabulated in Table 6. The parameters in Table 6 for the 1.0 $\mathrm{ms^{-2}}$ r.m.s. magnitude vibration are the same as those presented in Table 3. The frequencies at which the normalised apparent masses calculated from the models were greatest are also presented in Table 6. For the two models, the differences in the normalised apparent mass and phase between calculated values and measured values were less than about 0.06 and 3°. The responses of Model 2a at the four vibration magnitudes are compared with the mean, maximum and minimum values obtained in the experiment in Figs. 7 and 8.

The effect of vibration magnitude on model parameters was investigated statistically with the parameters obtained for individual subjects at different vibration magnitudes. For Model 2a, the

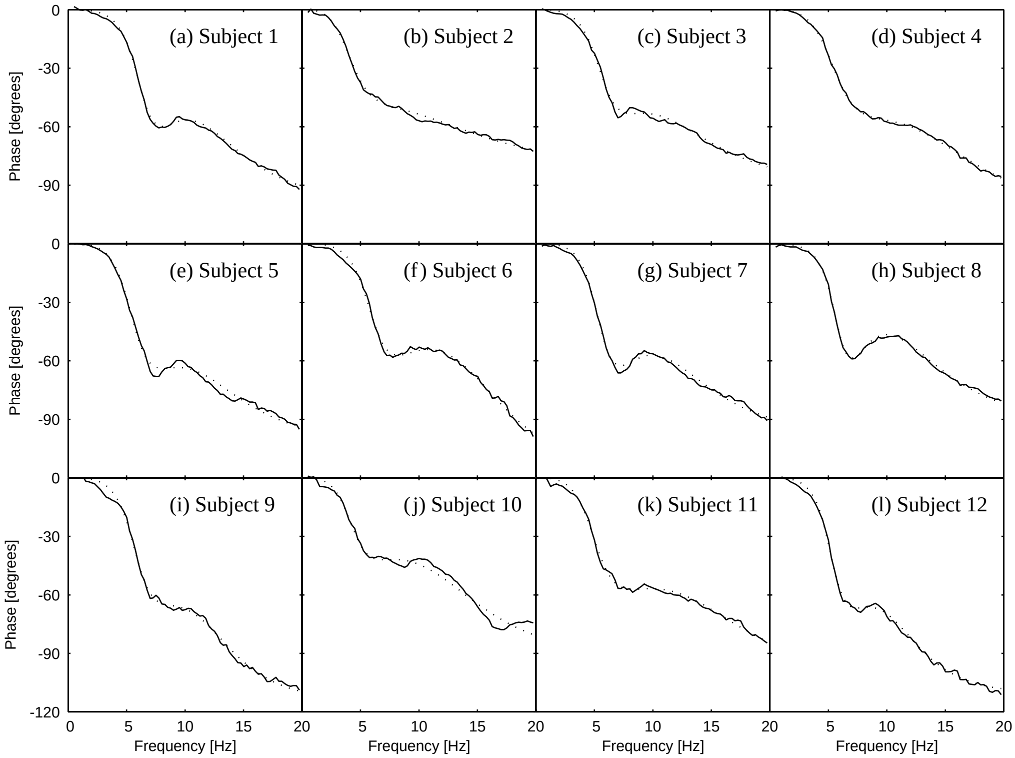

Fig. 6. Comparison between the measured phase of the apparent mass and the phase of the apparent mass of Model 2a with the parameters presented in Table 4 for 12 subjects in normal standing posture: ——, experiment; - - - -, model.

Table 6
Optimised model parameters of Model 2a and c for the mean normalised apparent masses of 12 subjects in a normal standing posture at four vibration magnitudes

Vibration magnitude (ms^{-2} r.m.s.)	Stiffness ($Nm^{-1}kg^{-1}$)		Damping ($Nsm^{-1}kg^{-1}$)		Mass (no unit)		Peak frequency (Hz)
	k_1	k_2	c_1	c_2	m_1	m_2	
Model 2a							
0.25	5.56×10^3	7.29×10^2	3.86×10^1	1.47×10^1	5.74×10^{-1}	4.17×10^{-1}	5.70
0.5	5.25×10^3	6.48×10^2	3.79×10^1	1.37×10^1	5.63×10^{-1}	4.11×10^{-1}	5.40
1.0	4.39×10^3	5.53×10^2	3.71×10^1	1.18×10^1	5.74×10^{-1}	3.94×10^{-1}	5.08
2.0	3.32×10^3	4.86×10^2	3.26×10^1	1.16×10^1	5.70×10^{-1}	4.04×10^{-1}	4.68
Model 2c							
0.25	2.67×10^3	1.16×10^3	2.41×10^1	2.08×10^1	3.23×10^{-1}	6.79×10^{-1}	5.74
0.5	2.60×10^3	1.03×10^3	2.34×10^1	1.99×10^1	3.20×10^{-1}	6.65×10^{-1}	5.44
1.0	2.37×10^3	8.49×10^2	2.48×10^1	1.65×10^1	3.45×10^{-1}	6.33×10^{-1}	5.11
2.0	1.63×10^3	7.71×10^2	1.92×10^1	1.64×10^1	3.04×10^{-1}	6.80×10^{-1}	4.72

Nominal corresponding parameters for a specific static mass can be obtained by multiplying the parameters shown by the static mass. Frequencies at which the calculated normalised apparent masses are greatest are also shown.

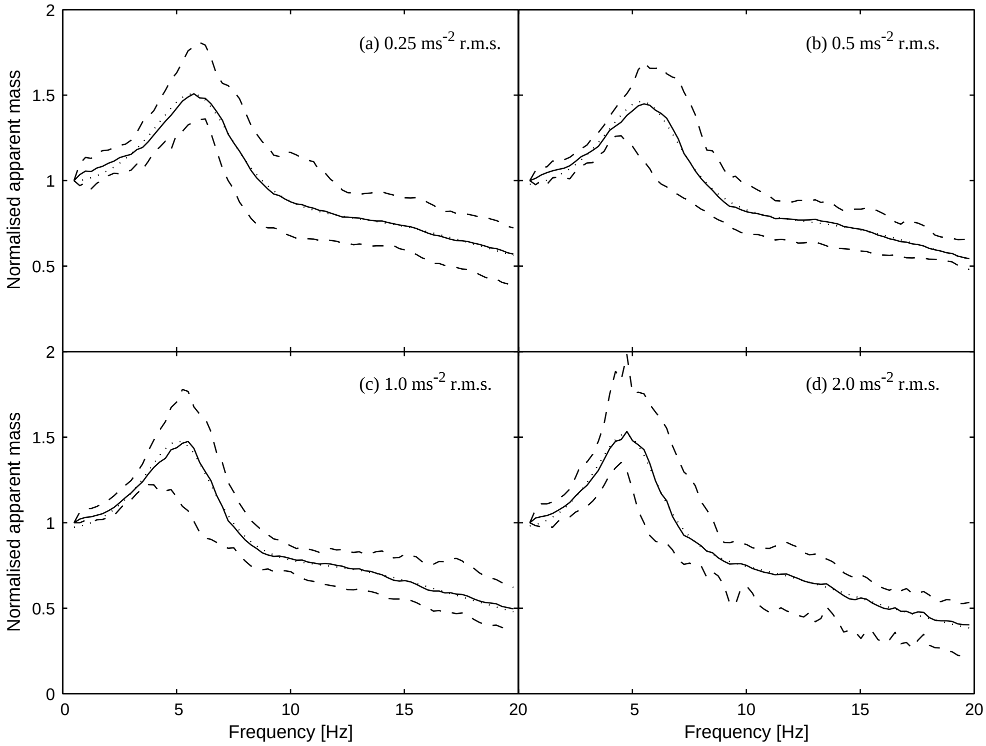

Fig. 7. Mean measured normalised apparent masses in normal standing posture and apparent masses calculated from Model 2a with parameters shown in Table 6 for four vibration magnitudes: ——, mean measured value; — —, maximum and minimum measured value; - - - -, model.

stiffness coefficients k_1 and k_2 decreased with each increase in vibration magnitude, except for the change from 0.25 to 0.5 ms^{-2} r.m.s. ($p < 0.05$, Wilcoxon matched-pairs signed ranks test). The damping coefficient, c_1, obtained with the vibration magnitude of 2.0 ms^{-2} r.m.s. was significantly less than those with the other three magnitudes ($p < 0.05$). The damping coefficient, c_2, decreased with increases in vibration magnitude from 0.5 to 1.0 ms^{-2} r.m.s. and from 1.0 to 2.0 ms^{-2} r.m.s. ($p < 0.05$). There were no significant changes in the masses m_1 and m_2 between different magnitudes, except between 0.5 and 2.0 ms^{-2} r.m.s.: m_1 was significantly less at 0.5 ms^{-2} r.m.s. than at 2.0 ms^{-2} r.m.s. while m_2 was significantly greater at 0.5 ms^{-2} r.m.s. than at 2.0 ms^{-2} r.m.s. ($p < 0.05$).

For Model 2c, one of the stiffness coefficients, k_1, optimised with the vibration magnitude of 2.0 ms^{-2} r.m.s. was significantly less than that with the other three vibration magnitudes ($p < 0.01$). The other stiffness coefficients, k_2, decreased with each increase in vibration magnitude, except for the change from 1.0 to 2.0 ms^{-2} r.m.s. ($p < 0.05$). There were no significant changes found in the damping parameter c_1 due to changes in vibration magnitude. The damping parameter c_2 at 0.25 and 0.5 ms^{-2} r.m.s. was significantly greater than at 1.0 and 2.0 ms^{-2} r.m.s. ($p < 0.05$). The mass parameters did not change significantly due to changes in vibration magnitude, except that the mass m_1 at 0.5 ms^{-2} r.m.s. was significantly greater than that at 2.0 ms^{-2} r.m.s. ($p < 0.05$).

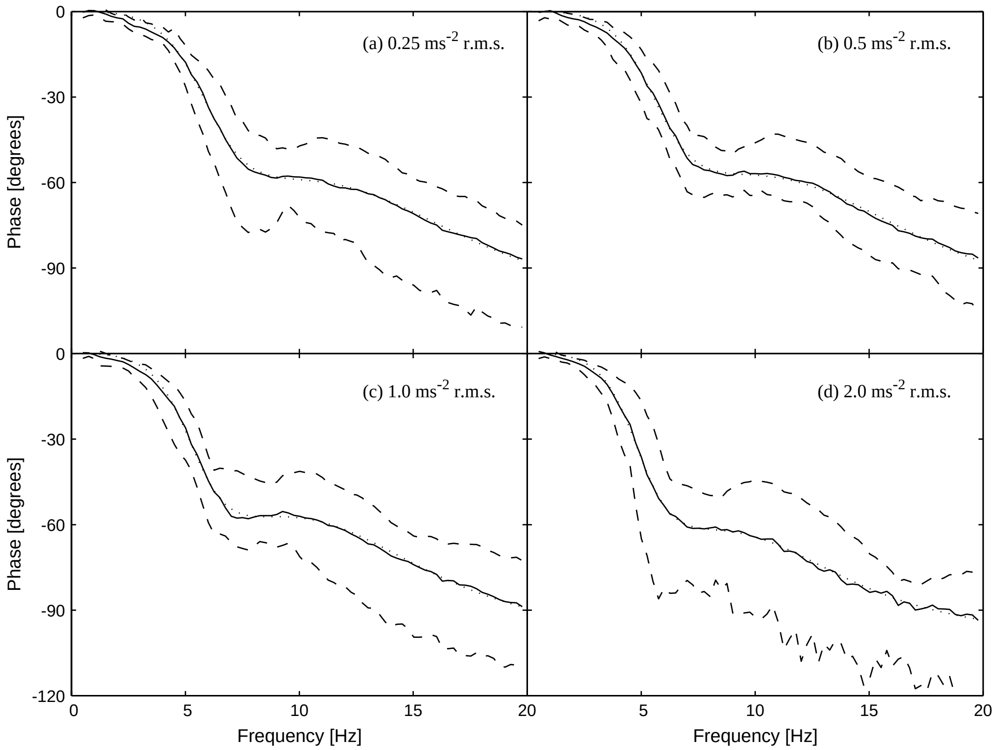

Fig. 8. Mean measured phases of the apparent mass in normal standing posture and phases calculated from Model 2a with parameters shown in Table 6 for four vibration magnitudes: ——, mean measured value; — —, maximum and minimum measured value; - - - -, model.

5.2. Effect of posture on model parameters

All model types shown in Fig. 1 were used to develop a mathematical model for the apparent mass in the 'legs-bent' posture and in the 'one-leg' posture because the characteristics of the apparent mass in these two postures observed in the experiment were different from those in the normal standing posture. Tables 7 and 8 present the optimised parameters obtained with the mean normalised apparent masses of 12 subjects in the legs bent posture and in the one leg posture, as defined in Table 2, measured at 1.0 ms^{-2} r.m.s. For the two d.o.f. models with a mass of m_0 for the support structure, 10% of the sum of the other two masses was assigned to m_0 because m_0 converged to a value less than 1% of the total mass in the parameter identification. Comparisons between the model responses and the experimental data are made in Figs. 9 and 10 for the legs bent posture and in Figs. 11 and 12 for the one leg posture.

The optimised model responses for the legs bent posture did not represent the experimental data as accurately as those for the normal standing posture (Figs. 9 and 10, compared to Figs. 2 and 3). The maximum differences between the mean responses measured in the experiment and the responses calculated from the models with single d.o.f., Model 1a and b, were about 0.2 for the normalised apparent mass and about 60° for the phase. For the models with two d.o.f., the model

Table 7
Optimised model parameters of all model types for the mean normalised apparent masses of 12 subjects in a legs bent posture

	Stiffness ($Nm^{-1}kg^{-1}$)		Damping ($Nsm^{-1}kg^{-1}$)		Mass (no unit)		
	k_1	k_2	c_1	c_2	m_0	m_1	m_2
Model 1a	2.79×10^2	—	1.04×10^1	—	—	8.33×10^{-1}	—
Model 1b	2.53×10^2	—	7.43×10^0	—	1.89×10^{-1}	7.19×10^{-1}	—
Model 2a	3.75×10^2	3.22×10^2	3.02×10^1	3.38×10^0	—	4.60×10^{-1}	4.88×10^{-1}
Model 2b	3.36×10^2	3.42×10^2	1.95×10^1	2.21×10^0	8.62×10^{-2}	4.49×10^{-1}	4.13×10^{-1}
Model 2c	2.52×10^2	1.39×10^3	7.31×10^0	1.55×10^1	—	7.17×10^{-1}	1.72×10^{-1}
Model 2d	2.52×10^2	7.86×10^2	7.25×10^0	6.06×10^0	8.23×10^{-2}	7.13×10^{-1}	1.10×10^{-1}

Nominal corresponding parameters for a specific static mass may be obtained by multiplying the parameters shown by the static mass.

Table 8
Optimised model parameters of all model types for the mean normalised apparent masses of 12 subjects in a one leg posture

	Stiffness ($Nm^{-1}kg^{-1}$)		Damping ($Nsm^{-1}kg^{-1}$)		Mass (no unit)		
	k_1	k_2	c_1	c_2	m_0	m_1	m_2
Model 1a	5.08×10^2	—	1.75×10^1	—	—	8.98×10^{-1}	—
Model 1b	4.79×10^2	—	1.44×10^1	—	1.06×10^{-1}	8.22×10^{-1}	—
Model 2a	6.66×10^2	6.07×10^2	2.90×10^1	8.81×10^0	—	5.13×10^{-1}	4.22×10^{-1}
Model 2b	5.30×10^2	3.95×10^2	1.85×10^1	3.39×10^0	8.60×10^{-2}	6.72×10^{-1}	1.88×10^{-1}
Model 2c	4.77×10^2	1.08×10^3	1.41×10^1	1.10×10^1	—	8.18×10^{-1}	9.85×10^{-2}
Model 2d	3.11×10^2	1.21×10^2	6.69×10^0	1.09×10^1	8.88×10^{-2}	4.85×10^{-1}	4.03×10^{-1}

Nominal corresponding parameters for a specific static mass may be obtained by multiplying the parameters shown by the static mass.

responses differed from the measured responses by less than about 0.12 for the normalised apparent mass and about 25° for the phase.

For the one leg posture, the model responses showed better agreement with the experimental data than for the legs bent posture (Figs. 11 and 12, compared to Figs. 9 and 10). The differences in the normalised apparent mass and phase between Model 1a and b and the experiment were about 0.1 and about 27°, at most. The maximum difference between two d.o.f. models and the experimental data were about 0.07 for the normalised apparent mass and about 16° for the phase.

With two d.o.f. models, the model parameters were also determined with some of the model parameters fixed at those obtained for the normal standing posture: the stiffness parameter k_1 and the damping parameter c_1 (case 1), the stiffness parameter k_2 and the damping parameter c_2 (case 2), or the stiffness parameters k_1 and k_2 and the damping parameters c_1 and c_2 (case 3) were optimised, while the rest of the parameters were fixed at those shown in Table 3. Fig. 13 shows the maximum error in the normalised apparent mass between the experiment and the response

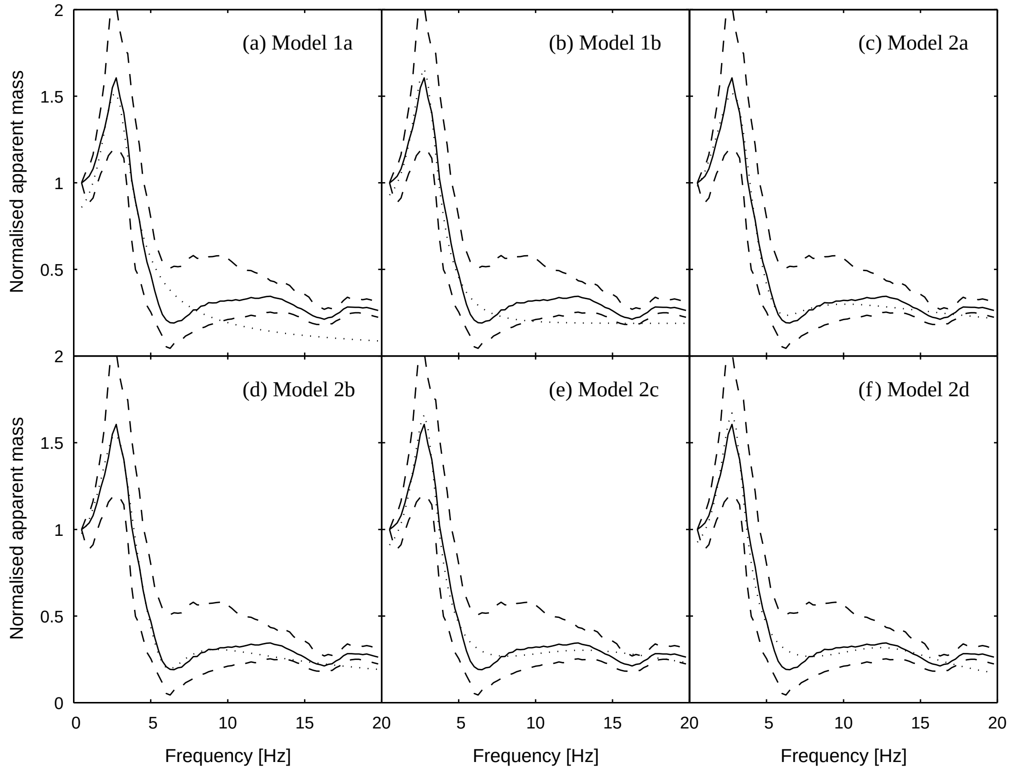

Fig. 9. Mean normalised apparent masses of 12 subjects in legs bent posture and the corresponding model responses for each model: ——, mean measured value; — —, maximum and minimum measured value; - - - -, model.

calculated by models with parameters obtained in cases 1–3 and when all parameters were optimised ('all' in Fig. 13). The maximum error in the phase of the apparent mass showed a similar trend to that in Fig. 13, although the data are not presented.

Fig. 13(a) shows that, for the legs bent posture, the normalised apparent mass calculated from Model 2a and b with all stiffness and damping parameters optimised were in reasonable agreement with the experimental data, with errors less than 0.13. The maximum error between the experimental measurements and Model 2c and d with all stiffness and damping parameters optimised was about 0.2. When either set of stiffness and damping parameters was optimised (i.e. case 1 or case 2), Model 2a and b with optimised k_1 and c_1 (case 1) showed smaller maximum error in the normalised apparent mass than Model 2a and b with optimised k_2 and c_2 (case 2), and Model 2c and d with optimised k_2 and c_2 (case 2) showed smaller maximum errors in the normalised apparent mass than Model 2c and d with optimised k_1 and c_1 (case 1).

For the one leg posture, the normalised apparent mass calculated by all two d.o.f. models with all stiffness and damping parameters optimised differed from the experimental data by 0.11 or less (Fig. 13(b)). Model 2a and b, when only k_1 and c_1 were optimised, also showed reasonable agreement with the experimental data with a maximum error of about 0.15.

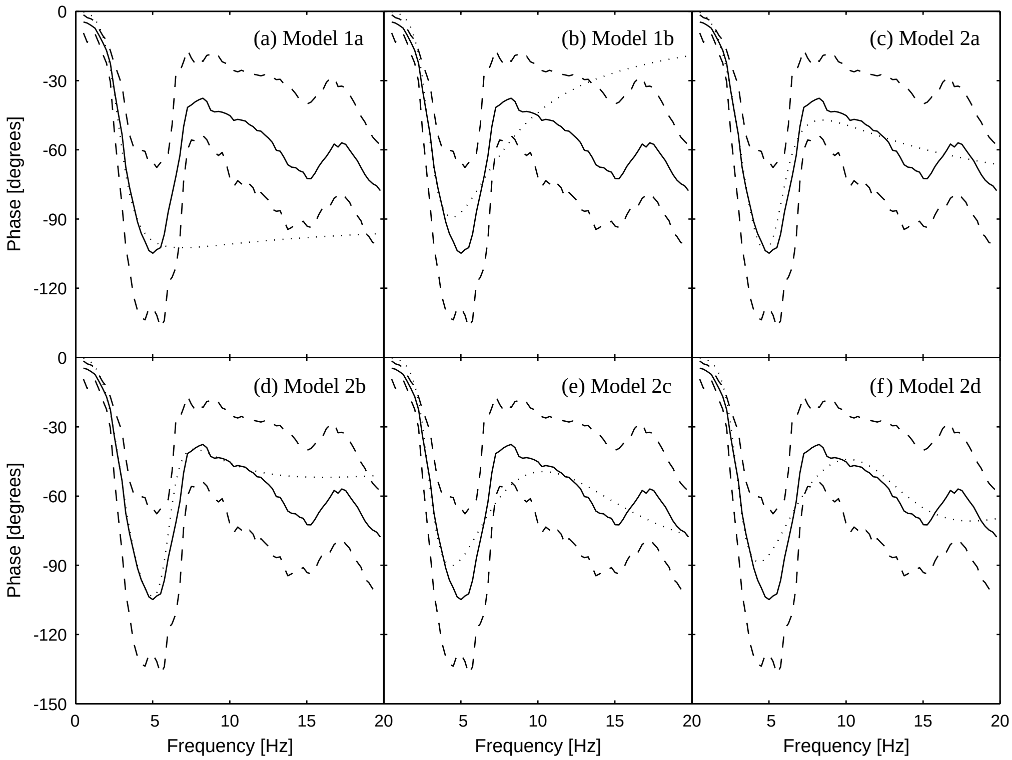

Fig. 10. Mean phase of the apparent masses of 12 subjects in legs bent posture and the corresponding model responses for each model. ——, mean measured value; — —, maximum and minimum measured value; - - - -, model.

6. Discussion

6.1. Model type

It was found that the two d.o.f. models represented the apparent masses of subjects in the three postures measured in the previous experiment [5] better than the single d.o.f. models. This finding is similar to previous conclusions with seated subjects as reported by Wei and Griffin [6]. The calculated responses of the two d.o.f. models with optimised parameters for each condition and each subject showed very good agreement with the measured apparent masses of standing subjects. The high agreement implies that more than two d.o.f. are not required to represent the apparent masses of standing subjects over the frequency range investigated.

The responses calculated by the models with a massless support showed better agreement with the experimental data than the responses calculated using models with a support structure having a mass of m_0, this trend was particularly clear in the phase data. There was no compliance between the support structure with a mass of m_0 and the vibrating base, which resulted in an increase in the phase at higher frequencies where the contributions of the other mass–spring–damper systems

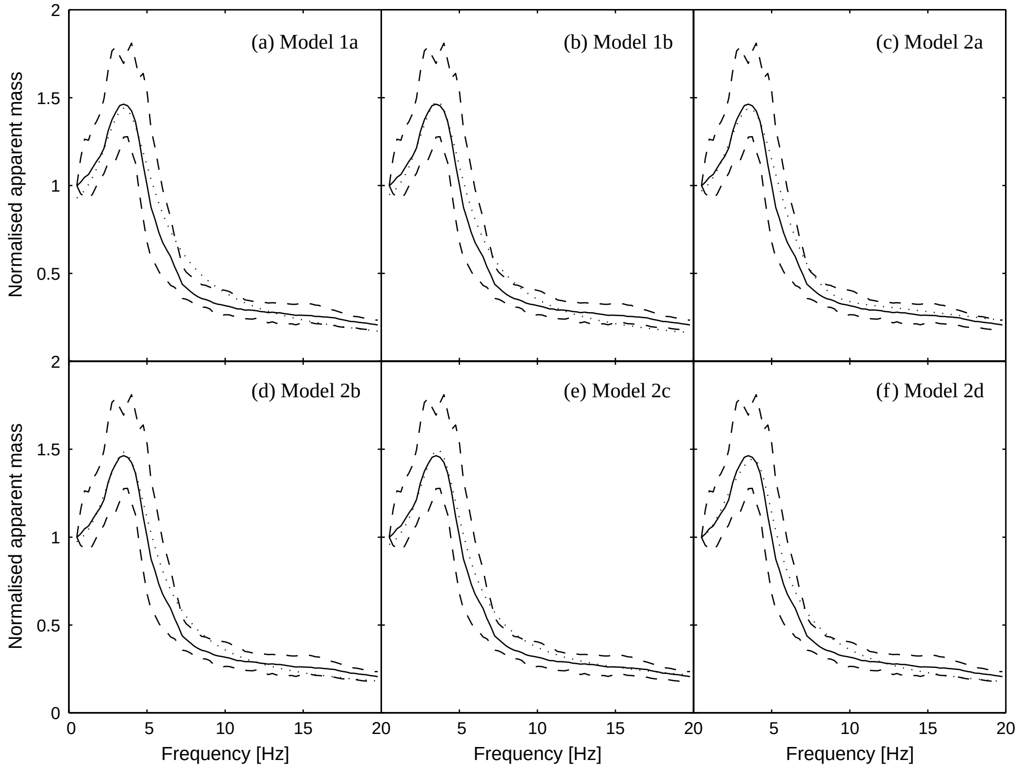

Fig. 11. Mean normalised apparent masses of 12 subjects in one leg posture and the corresponding model responses for each model: ——, mean measured value; — —, maximum and minimum measured value; - - - -, model.

were relatively small. This increase in the phase at higher frequencies was not observed in the experimental data, so the phase of the apparent mass calculated by the models with a support structure with a mass of m_0 differed at those frequencies. In addition, a reasonable value for the parameter of m_0 could not be obtained by the parameter identification method: as mentioned earlier, m_0 tended to converge to a very small value, less than 1% of the total mass. It can be concluded, therefore, that the models with a massless support structure as investigated in this study represent the apparent mass of subjects standing normally more reasonable than the models with a support structure having a mass of m_0. This is not consistent with the apparent masses of seated subjects: Wei and Griffin [6] recommended models with a support structure having mass. This is mainly because of the difference in the phase characteristics between the apparent masses of standing subjects and those of seated subjects. A support structure with a mass m_0 can be considered to represent the effect on responses in the frequency range investigated of vibration modes whose natural frequencies are higher than the highest frequency investigated. The effect of higher vibration modes on the apparent mass within the frequency range investigated may be different for standing and seated subjects. However, the physical mechanisms in the body that cause this effect are not reflected in the current models as they are not intended to represent the movement of any particular body parts.

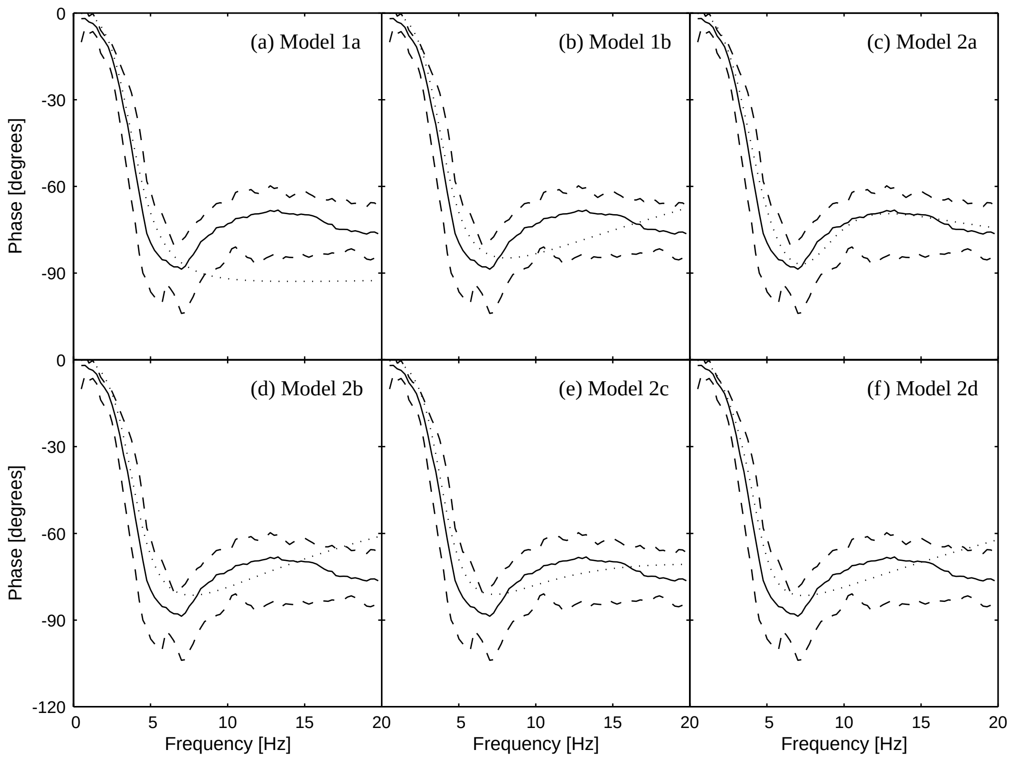

Fig. 12. Mean phase of the apparent masses of 12 subjects in one leg posture and the corresponding model responses for each model: ——, mean measured value; — —, maximum and minimum measured value; - - - - model.

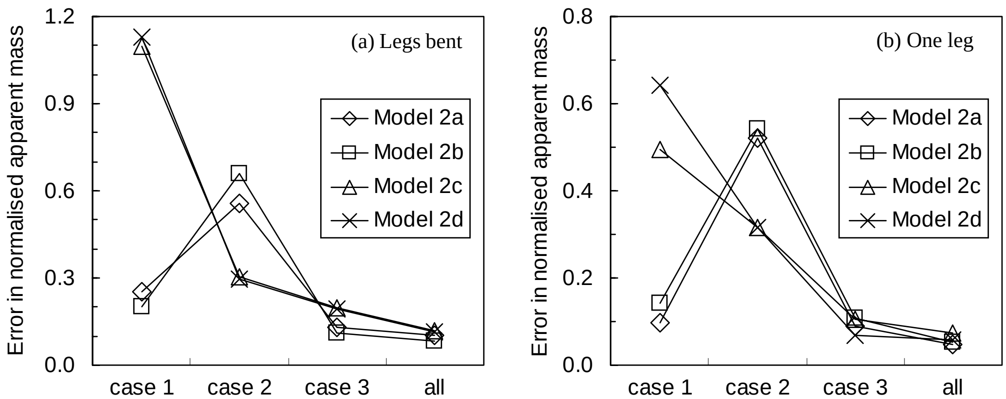

Fig. 13. Maximum error in normalised apparent mass between model and experiment for legs bent posture and for one leg posture; case 1: k_1 and c_1 were optimised; case 2:, k_2 and c_2 were optimised; case 3: k_1, k_2, c_1 and c_2 were optimised; all: all parameters were optimised. For cases 1–3, the rest of the parameters were fixed at those obtained for the normal posture.

6.2. Models for different conditions

In two d.o.f. models with two mass–spring–damper systems in series, such as Model 2a, each parameter in the model contributes to the properties of both vibration modes. However, in two d.o.f. models with two mass–spring–damper systems in parallel, such as Model 2c, each vibration mode is determined only by the dynamic behaviour of one of the two single d.o.f. systems in the model. This is because there is dynamic coupling between two mass–spring–damper systems in the models with these systems in series while there is no dynamic coupling in the models with parallel mass–spring–damper systems, which is obvious from Eqs. (5) and (7). With respect to the effect of vibration magnitude on the apparent mass, there were statistically significant decreases in the two stiffness parameters, k_1 and k_2, in Model 2a with each increase in vibration magnitude. However, in Model 2c, there were statistically significant decreases in only one of the two stiffness parameters, k_2, with each increase in vibration magnitude. The stiffness k_2 in Model 2c contributed to the principal resonance of the apparent mass at about 5 Hz, while the stiffness k_1 contributed to the broad peak at around 12 Hz. The statistically significant change found only in k_2 implies that the change in vibration magnitude had an effect mainly on the vibration mode responsible for the principal resonance of the apparent mass. For Model 2a, the statistically significant changes in both stiffness parameters with changes in vibration magnitude might have arisen because both parameters contributed to the principal resonance. The damping parameters tended to decrease with increases in vibration magnitude, although the changes in the damping parameters were not so clear as the changes in the stiffness parameters. The effect of vibration magnitude on the mass parameter was small. Two d.o.f. models with optimised stiffness and damping parameters and fixed mass parameters, therefore, may adequately represent the effect of vibration magnitude on the apparent mass.

All two d.o.f. models investigated in this study provided reasonable agreement with the experimental data after optimising all model parameters. When the mass parameters were fixed at those obtained for the apparent mass in the normal standing posture, those models with all stiffness and damping parameters optimised also showed good agreement with the experimental data for the legs bent posture and the one leg posture. Therefore, if all stiffness and damping parameters are optimised, mass parameters may not need to be optimised so as to represent the effect of postural changes from the normal standing posture to the legs bent posture or the one leg posture.

In the legs bent posture, differences in the apparent mass between the model response and the experimental data appeared to be greater than those for the normal standing posture and the one leg posture. However, greater differences occurred at frequencies where the normalised apparent mass was relatively small, mostly less than 0.3, so the differences were not important if the representation of the principal resonance is of primary interest.

7. Conclusions

Six alternative linear lumped parameter models for the representation of the apparent mass of the standing human body exposed to vertical whole-body vibration have been investigated. In three different standing postures and at four vibration magnitudes, two d.o.f. models with a massless support, either with two mass–spring–damper systems in series or with two mass–spring–damper systems in parallel, showed good agreement with measured apparent masses and phases. For the normal standing posture, the differences between the model response and the experimental data were less than 0.1 in normalised apparent mass (i.e., corresponding to less than 10% of the static mass) and less than 5° in phase. The model parameters obtained by fitting the mean measured apparent masses of all subjects were similar to the means of all sets of parameters obtained by fitting the individual apparent masses. Although the best representation of the effects of vibration magnitude and postural changes were obtained with apparent mass models with all parameters optimised, the effects were well represented by changing only the stiffness and damping parameters while fixing the masses at the values obtained for the normal standing posture at an intermediate vibration magnitude.

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